Disjoint Events vs. Independent Events

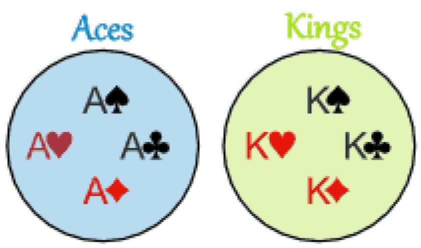
Disjoint events are events that do not have any outcomes in common, in other words there is no overlap:

Example: Drawing a single card from a deck

A = get an ace

B = get a King

These events are disjoint, because there is no overlap.

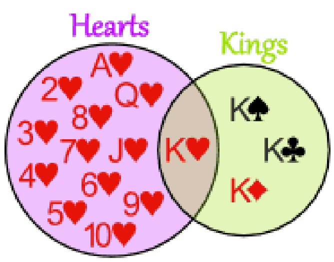


Example: Drawing a single card from a deck

A = get a Heart

B = get a King

These events are NOT disjoint, because there IS overlap.



Independent events are events such that knowing whether or not one event occurs does NOT affect the probability of another event occurring.

Event B is independent of Event A if: $P(B \mid A) = P(B)$

Key: two or more (nonimpossible) events **CANNOT be both disjoint and independent**. The reason is that: if one of the events occurs, it means that the other event cannot occur. The occurrence of one event affects the probability of the other.

Example: Drawing a single card from a deck

A = get an ace ➔ P(A) = 4/52 = 1/13

B = get a King ➔ P(B) = 4/52 = 1/13

We already said above that these are disjoint events. However, they are NOT independent!

What is P(B | A) ? In other words, what is P(get a King | got an ace) ?

It's 0! If I know that I got an ace, then there is ZERO chance that I got a King.

P(B | A) = 0, which is different from P(B) = 1/13, therefore they are NOT independent.

Look at another example from p. 8 of the Chapter 5 Handouts:

Example: Rolling two dice

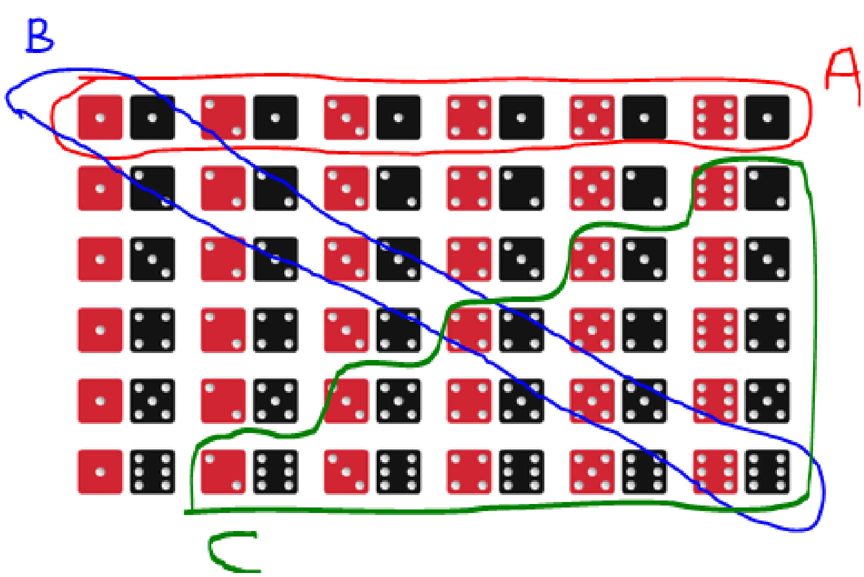
A = event that the black die is a 1

B = event that both dice are displaying the same number

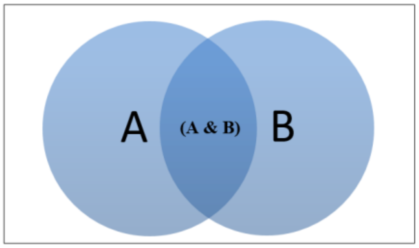
C = event that the sum of the dice is more than 7

1. A & B: are they disjoint? **No.**

2. A & C: are they disjoint? **Yes.**



A and B are NOT disjoint, because they have overlap, like this:



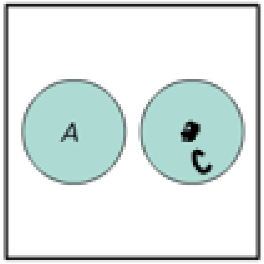
Are they independent?

P(B) = 6/36 = 1/6

P(B | A) = P(both dice same | black die is a 1) = 1/6

The probabilities are the SAME, therefore the events are independent!

A and C ARE disjoint, because they do not have overlap, like this:



Are they independent? (Remember, we already know that they cannot be, this is not possible).

P(C) = 15/36

P(C | A) = P(sum more than 7 | black die is 1) = 0

They are **NOT independent**. The probability of C changes based on the knowledge that A already happened.

Note: this works both ways. You could also show that P(A | C) ≠ P(A).

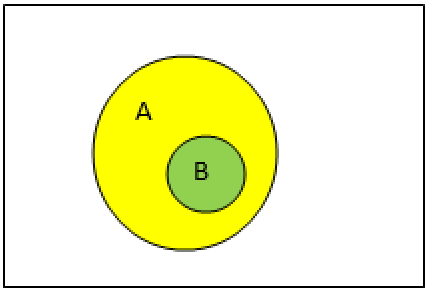
Look at one more example, from p. 21 of the Chapter 5 Handouts.

<u>Example</u>: Face cards, pick a random card

Let: A = get a face card (Jack, Queen or King)

B = get a Queen

In this case, there is overlap, in fact B is a subset of A, so a Venn diagram would look like this:



So they are clearly **NOT disjoint**. Are they independent?

P(B) = P(Queen) = 4/52 = 1/13

P(B | A) = P(Queen | it is a face card) = 4/12 = 1/3

P(B | A) ≠ P(B), therefore they are **NOT independent**.

To summarize the cases above:

Example:	**Disjoint?**	**Independent?**
A = get ace, B = get King	Yes	No
A = black die ‘1’, B = both dice same	No	Yes
A = black die ‘1’, C = sum > 7	Yes	No
A = get a face card, B = get a Queen	No	No

It would appear that the only case that is NOT possible is the case where they are both disjoint and independent, because as discussed above, that case is impossible.